* was in a hurry - please let me know if you find errors

NAME:

MATH-8 TEST Unit 3 (Intro to Trig Identities, Equations and Inverse Trig)
SAMPLE

100 points **NAME:** ____________

- **Notebook should be turned in <u>before</u> test. It will not be accepted after.**
- **Phones must be turned OFF and put away. Any visible phone (smart watch, headphones, ipad etc.) will result in a grade F .**
- **No scratch paper or notes.**
- **No graphing calculator.**
- **No credit will be given for solutions if work is not shown.**
- **I expect clear and legible presentations .**

This test is in two parts. On part one, you may not use a calculator; on part two, a calculator is necessary. When you complete part one, tear it off and place it at the front of your desk, I will collect it. Once you have turned in part one, you may not go back to it.

PART ONE - NO CALCULATORS ALLOWED

(1) Find each of the following: (Note: answers to inverse trig. problems should be in radians, not degrees)

(a) $\sin^{-1}\left(\frac{-1}{2}\right) =$ $\frac{-\pi}{6}$

(b) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\frac{5\pi}{6}$

(c) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ $\frac{\pi}{4}$

(d) $\sin\left(\frac{2\pi}{3}\right) =$ $\frac{\sqrt{3}}{2}$

(e) $\tan^{-1}\left(\sqrt{3}\right) =$ $\frac{\pi}{3}$

(f) $\sin^{-1}(1) =$ $\frac{\pi}{2}$

(g) $\cos^{-1}(2) =$ undefined

(h) $= \tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ $\frac{-\pi}{6}$

(2) HOW MANY solutions does each of the following equations with the given restrictions on θ have? (Do not need to solve, just tell how many solutions there would be.)

(a) $\sin\theta = -1/7;\ \ 0 \le \theta \le 2\pi$ 2

(c) $\sin\theta = -1/7$ infinitely many

(b) $\theta = \sin^{-1}(-1/7)$ 1

(d) $\sin\theta = -1/7;\ \ 0 \le \theta \le \frac{\pi}{2}$ none

(3) The domain of $f(x) = \cos^{-1} x$ $[-1, 1]$

(4) The range of the function $f(x) = \sin^{-1} x$ $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(5) The period of the function $f(x) = \tan x$ is π.

(6) The vertical asymptotes of $f(x) = \sec(5x)$ are located at: $x = \frac{\pi}{10} + \frac{\pi}{5}k,\ k \in \mathbb{Z}$

denom = 0

$\cos 5x = 0$

$5x = \frac{\pi}{2} + \pi k$

$x = \frac{\pi}{10} + \frac{\pi}{5}k$

NAME: ______________

MATH 8 Test 3 – SAMPLE
PART TWO - CALCULATORS ALLOWED (no graphing calc.)
Show your work on this paper. EXACT answers are expected unless otherwise specified.
Fill in the blanks with the most appropriate, simplified answer.

Fill in the blanks. (2 points each)

(1) Give an identity for $\sin 2\theta =$ $2\sin\theta\cos\theta$

(2) Give an identity for $\cos(\alpha+\beta) =$ $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

(3) Give an identity for $\sin(\theta/2) =$ $\pm\sqrt{\frac{1-\cos\theta}{2}}$

(4) What is domain of $f(x) = \cos^{-1}(x)$? $[-1, 1]$

(5) Find all asymptotes for $f(x) = 3\tan\left(\frac{\pi}{5}x\right)$ $x = \frac{5}{2} + 5k$

denom $= 0 \Rightarrow \cos\frac{\pi}{5}x = 0$

$\frac{\pi}{5}x = \frac{\pi}{2} + \pi k$

$x = \frac{5}{2} + 5k$

(6) What is the range of $f(x) = \tan^{-1}(x)$? $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(7) Solve for $0 \le x < 2\pi$: $\sin x = -1$ $\frac{3\pi}{2}$

(8) Using identities, find the exact, simplified value of: (points each)
(you must show work, for credit)

(a) $\sin\left(\frac{7\pi}{12}\right)$ $\frac{\sqrt{2}+\sqrt{6}}{4}$

(b) $\cos(157.5°)$ $\frac{-\sqrt{2+\sqrt{2}}}{2}$

157.5° in Q2

either use

$\sin\frac{7\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \frac{\sqrt{2}}{2}\frac{1}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

or

$\sin\frac{7\pi}{12} = \sin\left(\frac{7\pi/6}{2}\right) = \sqrt{\frac{1+\sqrt{3}/2}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$

$\cos 157.5° = \cos\frac{315°}{2}$

$= -\sqrt{\frac{1+\cos 315°}{2}}$

$= -\sqrt{\frac{1+\sqrt{2}/2}{2}} = \frac{-\sqrt{2+\sqrt{2}}}{2}$

(9) Simplify exactly:

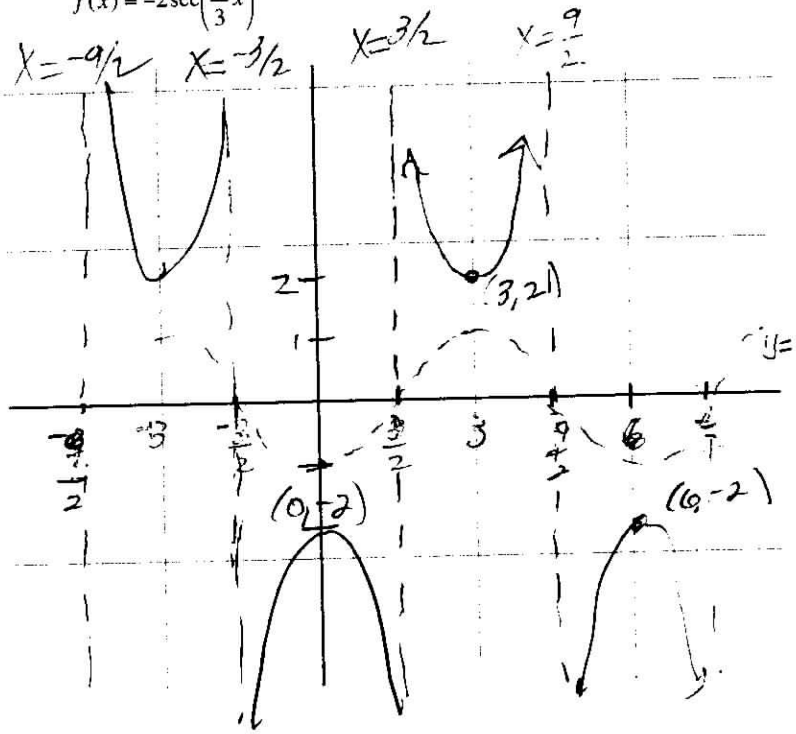
(a) $\cos(\sin^{-1}(-2/5)) =$

(b) $\sin(2\cos^{-1}(1/4)) =$

— Not covered yet —
not on this test

(10) Sketch the following graph. (clearly show scale, graph at least one period, show location of any asymptotes, label 2 points on graph)

$f(x) = -2\sec\left(\frac{\pi}{3}x\right)$



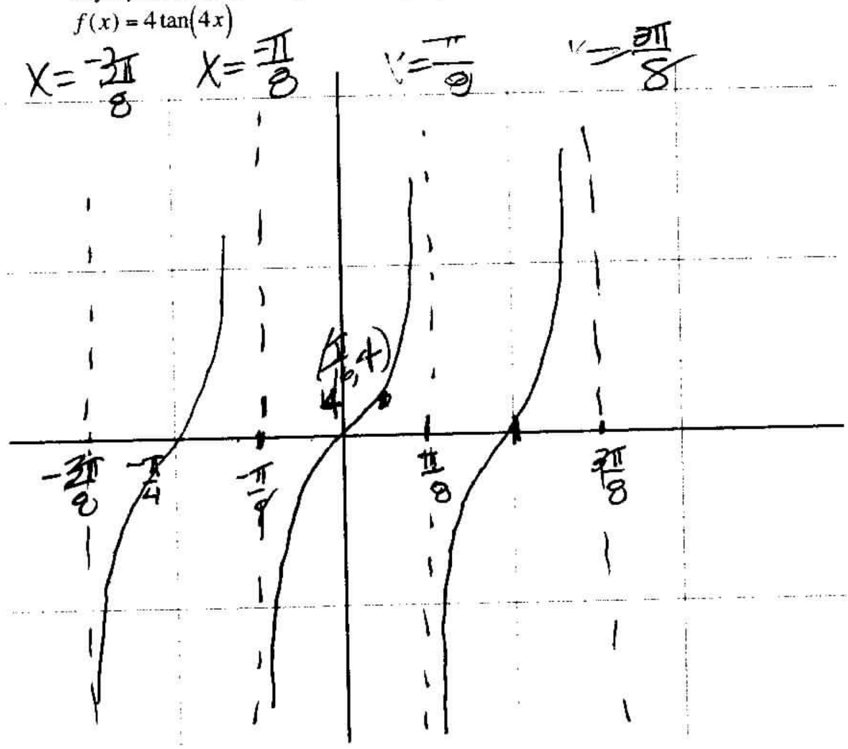
graph $y = -\cos\left(\frac{\pi}{3}x\right)$
then use reciprocal
and vertical stretch by 2

period: $\frac{2\pi}{\pi/3} = 6$

$\frac{1}{4}$ period $= \frac{3}{2}$

(11) Sketch the following graph. (clearly show scale, graph at least one period, show location of any asymptotes, label 2 points on graph)

$f(x) = 4\tan(4x)$

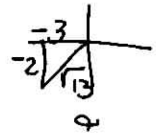


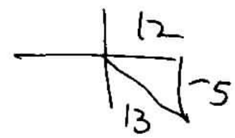
Asymptotes when
$\cos 4x = 0$
$4x = \frac{\pi}{2} + \pi k$
$x = \frac{\pi}{8} + \frac{\pi}{4}k$
period is $\frac{\pi}{4}$

check ... period $= \frac{\pi}{4}$

(12) Given $\tan\alpha = 2/3$, α in the third quadrant, and $\cos\theta = 12/13$, $\frac{3\pi}{2} < \theta < 2\pi$

Find:





a) $\sin(\alpha - \theta)$

$\sin\alpha\cos\theta - \cos\alpha\sin\theta$

$\frac{-2}{\sqrt{13}}\frac{12}{13} - \frac{-3}{\sqrt{13}}\left(\frac{-5}{13}\right) = \frac{-39}{13\sqrt{13}} = -\frac{3}{\sqrt{13}}$

b) $\cos(\theta/2)$

$\frac{3\pi}{4} < \frac{\theta}{2} < \pi$ (Q2)

$\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}} = -\sqrt{\frac{1+12/13}{2}} = -\frac{5}{\sqrt{26}}$

c) $\tan(2\alpha)$

$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\cdot\frac{2}{3}}{1-\frac{4}{9}} = \frac{\frac{4}{3}}{5/9} = \frac{4}{3}\cdot\frac{9}{5} = \frac{12}{5}$

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(13) Prove the following identity. Presentation should be very clear

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

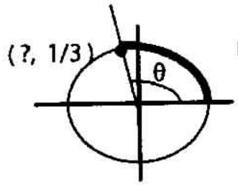
$\frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan\theta}{\sec^2\theta} = \frac{2\tan\theta}{\frac{1}{\cos^2\theta}} = 2\tan\theta\cdot\cos^2\theta$

$(\tan^2\theta + 1 = \sec^2\theta)$

$= 2\frac{\sin\theta}{\cos\theta}\cos^2\theta = 2\sin\theta\cos\theta = \sin 2\theta$

$\therefore \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

(14) Given the following information about θ,



unit circle

$x^2+y^2=1$

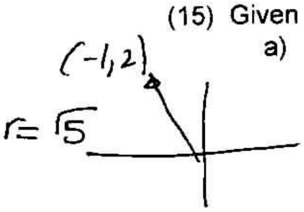
$x^2+\frac{1}{9}=1$

$x^2=\frac{8}{9}$ $\quad x=\frac{-2\sqrt{2}}{3}$

$\tan\theta=\frac{y}{x}=\frac{1/3}{\frac{-2\sqrt{2}}{3}}=-\frac{1}{2\sqrt{2}}=\frac{-\sqrt{2}}{4}$

Find a) $\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}=\frac{2\left(-\frac{\sqrt{2}}{4}\right)}{1-\frac{2}{16}}=\frac{-4\sqrt{2}}{7}$ b) $\cos\left(\frac{\theta}{2}\right)=\sqrt{\frac{1+\cos\theta}{2}}=\sqrt{\frac{1-2\sqrt{2}/3}{2}}=\sqrt{\frac{3-2\sqrt{2}}{6}}$

(15) Given that P(-1,2) lies on the terminal side of θ, find



a) $\cos(2\theta)$.

$2\cos^2\theta-1$

$2\left(\frac{-1}{\sqrt{5}}\right)^2-1$

$\frac{2}{5}-1=3/5$

b) θ $\tan^{-1}(-2)+\pi$

OR

$\pi-\tan^{-1}(2)$

(16) Solve the following equations exactly for $0\le\theta\le2\pi$.

(a) $\cos\theta=\frac{1}{5}$ (b) $\sin\theta=0.8$ c) $\tan\theta=-7$

ref $=\tan^{-1}7$

$\theta=\cos^{-1}\frac{1}{5},\ 2\pi-\cos^{-1}\frac{1}{5}$ $\quad\theta=\sin^{-1}0.8,\ \pi-\sin^{-1}0.8$ $\quad\theta=\pi-\tan^{-1}7,\ 2\pi-\tan^{-1}7$

(17) Solve for $0\le x<2\pi$: $\sqrt{3}\tan(2x)+1=0$ two times around

$\tan 2x=-\frac{1}{\sqrt{3}}$

$2x=\frac{5\pi}{6},\frac{11\pi}{6},\frac{17\pi}{6},\frac{23\pi}{6}$

$x=\frac{5\pi}{12}\ \frac{11\pi}{12}\ \frac{17\pi}{12}\ \frac{23\pi}{12}$

(18) Find all solutions: $4\cos\left(\frac{x}{3}\right)=-4$

$\cos\frac{x}{3}=-1$

$\frac{x}{3}=\pi+2\pi k$

$x=3\pi+6\pi k$ $\quad k\in\mathbb{Z}$

Find all solutions to the following equations.

(19) $2\sin^2(x) - \cos(2x) = 0$

$$2\sin^2 x - (1 - 2\sin^2 x) = 0$$

$$2\sin^2 x - 1 + 2\sin^2 x = 0$$

$$4\sin^2 x = 1$$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm \frac{1}{2}$$

$$x = \frac{\pi}{6} + \pi k, \ \frac{5\pi}{6} + \pi k$$

$$k \in \mathbb{Z}$$

(20) $4\cos^2 x - 2 = 0$

$$\cos^2 x = \frac{2}{4} = \frac{1}{2}$$

$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{4} + \pi k, \ \frac{3\pi}{4} + \pi k$$

$$k \in \mathbb{Z}$$

SOLVE the following equations: $0 \le x < 2\pi$

(21) $\sin(3x) - \sin(2x) = 0$

Sum to product

$$\sin\alpha - \sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\sin 3x - \sin 2x = 0$$

$$2\sin\left(\frac{x}{2}\right)\cos\left(\frac{5x}{2}\right) = 0$$

$$\sin\frac{x}{2} = 0 \qquad \cos\frac{5x}{2} = 0$$

$$\frac{x}{2} = \pi k \qquad \frac{5x}{2} = \frac{\pi}{2} + \pi k$$

$$x = 2\pi k \qquad x = \frac{\pi}{5} + \frac{2\pi k}{5}$$

In $[0, 2\pi)$

$$0, \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$$

(22) $\sec^2 x - 3\tan x = -1$

$$\tan^2 x + 1 - 3\tan x = -1$$

$$\tan^2 x - 3\tan x + 2 = 0$$

$$(\tan x - 2)(\tan x - 1) = 0$$

$$\tan x = 2 \qquad \tan x = 1$$

$$x = \tan^{-1} 2, \ \pi + \tan^{-1} 2, \ \frac{\pi}{4}, \ \frac{5\pi}{4}$$